Issue 53 – 15/06/21

Extension Activity

Exponential growth 2: real-life lessons from the COVID-19 pandemic

Activity 1 – The invention of chess

"One grain of rice, representing the first square of a chessboard. Two grains for the second square. Four grains for the next. Then eight, 16, 32... doubling for each successive square until the 64th and last square is counted."

Task: Find out how many grains of rice are on the 64th square:

Square	1	2	3	4	5	6	7	64	x
Number of rice grains	1								
Rice grains as a power of 2									

Student information for the discussion:

- Assume a weight of 30g per 1000 grains of rice.
- The annual global rice harvest in recent years has been about 500 million tonnes.

Activity 2 – Infectious diseases

Part 1: COVID-19 spread without protective measures

Student information:

To follow the course of a disease, we need to know how many people, on average, are infected by an infected person if no precautions are taken. This 'basic reproduction number', called R_0, is about four for COVD-19. If mutations occur that make the virus more contagious, the R_0 value increases accordingly. For polio, the R_0 value is six; and for measles, it is between 12 and 18.

In addition to R_0, the time, D, during which the infected person is infectious also plays a role. For COVID-19, D is about five days.

Task: For COVID-19, on average, one infected person infects four other people within five days. Complete the table and find a formula to calculate the number of newly infected people at time *x*.

Time in days	0	5	10	20	30	40	*x*
Number of newly infected people	1						

Draw a graph using a spreadsheet of your choice or use the Excel spreadsheet provided (Exponential Growth.xlsx) to find out how long it takes for 4 (→16→64) newly infected people to become 8 (→32→128) newly infected people – this is the doubling time.

Your measured doubling time is: ______

Your calculated doubling time is: ______

Part 2: Containment of the COVID-19 pandemic

Student information:

Measures like physical distancing and the use of face masks cause the infection to spread more slowly. This is expressed by the 'effective reproduction number', *R*.

Containment measures will reduce the *R* value. If mobility, and thus, the probability of contact and infection, is reduced by *X*%, then the effective reproduction number would be defined as $R = (1 - X/100) \times R_0$. This means that if mobility is restricted by 80%, then *R* is reduced to 20% of R_0. The use of a face mask to diminish aerosols also reduces the probability of infection by *Y*% and affects the *R* value. Together, these two measures result in $R = (1 - X/100) \times (1 - Y/100) \times R_0$.

Task: Use the Geogebra applet to find out more about how to contain the spread of COVID-19. With the help of sliders, you can modify the effects of physical distancing and face masks. You can also change the time at which each or both measures are implemented. You will find the applet here: https://www.geogebra.org/m/qavutkx5.

Investigate the following questions:

1. How does the timing of the containment measures change the course of the graph?
2. How do the following containment measures affect the doubling time if applied from the beginning?
    - only physical distancing with 50% effectivity
    - physical distancing plus use of face masks with 50% effectivity

Calculated doubling time with physical distancing only: _____

Calculated doubling time with physical distancing and face masks: ______

Part 3: COVID-19 spread in a more realistic setting – herd immunity

Student information:

The percentage of people who are immune (through vaccination or recovery from the disease), p_I, in the total population also affects the R value. Taking this effect into account is more complicated because this group becomes larger over time. At the beginning of the pandemic, p_I is zero, whereas at the end, when everybody is immune, the value is 100. The way in which p_I is used to calculate R is the same as that for physical distancing or face masks: $R = (1-p_I/100) \times R_0$.

Find out the minimum value of p_I required to contain the spread of the virus, i.e., to reduce the R value to one.

Compare the percentage of the population that needs to be immunized to achieve herd immunity for the following diseases:

COVID-19 ($R_0 = 4$):

Measles ($R_0 = 15$):

Polio ($R_0 = 6$):