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Exponential growth 2: real-life lessons from the COVID-19 pandemic

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The COVID-19 pandemic has shone a spotlight on exponential growth. This provides an opportunity to teach this tricky concept in a real-world context.

The extremely rapid spread of the COVID-19 virus is not as surprising as it may first seem. It is a consequence of exponential growth in the propagation of the virus, which is difficult to grasp intuitively.

The activities described below are designed for students aged 14–16 and can be completed in 45 minutes.

If a simpler approach seems more appropriate for your class, you can also use elements from the accompanying article for 11–13 year olds, Exponential growth 1: learn the basics from confetti to understand pandemics. That article also includes a brief comparison of linear and exponential growth. In the course of these activities, students will learn about different types of exponential growth and how to classify them using the *R* value. In the context of the COVID-19 pandemic, they will identify how growth can be decelerated and they will use a simulation to explore which actions affect the course of the pandemic.

Calculations for the tables can be simplified using a calculator or an Excel spreadsheet. Tables and coordinate systems for completion are on the worksheet.

Activity 1 – The invention of chess

A legend about the invention of chess reveals how quickly numbers can skyrocket with exponential growth. It tells the story of the Indian king Shihram, who supposedly lived in the third or fourth century AD and tyrannized his people, plunging his country into misery. To draw the king's attention to his faults without inflaming his anger, the grand vizier Sissa ben Dahir invented a game in which the king, as the most important piece, could do nothing without the help of other pieces. The teachings of chess made a strong impression on Shihram. He mellowed and widely publicized the game of chess. As thanks for the vivid life lesson and entertainment, he granted a wish to the grand vizier, who asked for only:

"One grain of rice, representing the first square of a chessboard. Two grains for the second square. Four grains for the next. Then eight, 16, 32... doubling for each successive square until the 64th and last square is counted."

The king was impressed with the apparent modesty of the request, and he immediately granted it. Why was this a huge mistake?

Materials

- The accompanying worksheet
- A calculator

Procedure

1. Ask students if they think the grand vizier was really that modest, or how many grains of rice are on the 64th square of a chessboard. The table on the worksheet can help.
2. Have them find a common rule to calculate the number of rice grains on the xth square. The table should look like this:

Square	1	2	3	4	5	6	7	64	x
Number of rice grains on that square	1	2	4	8	16	32	64	9.2×10^{18}	
Calculation	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^{63}	$2^{(x-1)}$

Discussion

Let the students read the background information on the worksheet about the mean annual global rice harvest and let them discuss the amount of rice the grand vizir asked for. The increase in the number of grains of rice from one square to the next is not constant, as in linear growth, but increases with the number of grains of rice on the preceding square. If the increase depends on the current quantity and is therefore not constant, it is called exponential growth. If the current quantity is small, the increase is also small, so the quantity grows slowly. However, the quantity still increases and so does the increment, which can then become very large very quickly.

Activity 2 – Infectious diseas es

In an infectious disease, the numbers of newly infected people can grow as rapidly as the quantities of rice on the squares of the chessboard in Activity 1. The spread of infectious diseases often follows a similar pattern, which can be described by only a few parameters.

Materials

- The accompanying worksheet
- A calculator
- A computer, tablet, or smartphone with an internet browser and spreadsheet

Part 1: Spread of COVID-19 without protective measures

1. Have the students read the background information on their worksheet about the R_0 and D values for the COVID-19 infection.
2. Ask the students to complete the table on the worksheet to track the runaway spread of COVID-19 and find a formula for calculating the number of newly infected people at time x. It should then look like this:

Time in days	0	5	10	20	30	40	x
Number of newly infected people	1	4	16	256	4096	65 536	$4^{(x/5)}$

3. Have the students draw a graph using a spreadsheet of your choice or use the Excel spreadsheet provided. It may look like this:

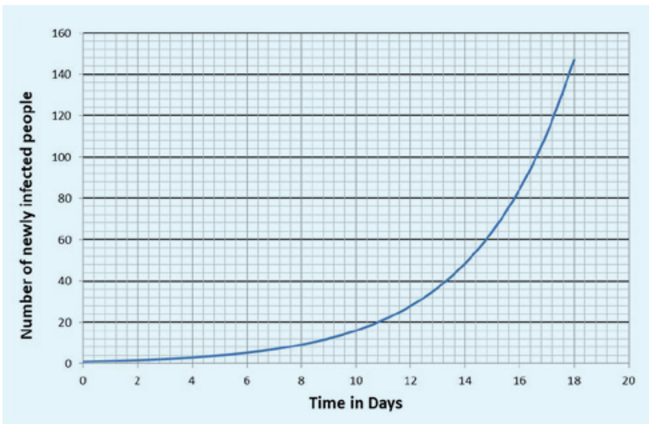

Image courtesy of Wolfgang Vieser

4. Have the students determine from the graph how long it takes for the number of newly infected people to double. How long before 4(→16→64) newly infected people become 8 (→32→128) newly infected people? The doubling time should be 2.5 days.
5. Derive the exact value of the doubling time together with the students. It may look like this:
   - At what time, x, did the number of newly infected people double from the initial value of one to a value of two? The corresponding equation to solve is $4^{(x/5)}$ **= 2**.
   - Ask the students to which number they have to raise four to produce two. Alternatively, you could ask which mathematical operation can be performed with four to get two (not subtract or divide). The answer should be to take the square root of four to get two or raise four to the power of 0.5.
   - The above equation thus becomes $x/5 = 0.5$
   - Therefore $x = 2.5$ so the doubling time is 2.5 days.
6. Let the students guess how many newly infected people to expect on day 50 and then have them calculate the value. There should be just over 1 million people newly infected on day 50 if no protective measures are taken.

Discussion

To derive the exact doubling time in the above case, it was only possible to use the square root because of the special base, four. Ask the students how to derive the doubling time in general.

In the general case, one must refer to the logarithm. If students are not yet familiar with the concept of logarithms you may introduce them as follows.
Important background information: Just like the root symbol is used to solve the equation $x \times x = 2$, to which the solution is $x = \pm\sqrt{2} \sim 1.414$, symbolic notation is used to solve the equation $2^x = 6$, which is $x = \log_2 6$ (logarithm of six to base two). The numerical value can be achieved with the help of a pocket calculator: $\log_2 6 \sim 2.585$.

The exact value of the doubling time, T1/2, can be derived as follows:

$$4^{x/5} = 2$$

$$\frac{x}{5} = \log_4 2$$

$$x = 5 \times \log_4 2 = 2.5$$

$$T_{1/2} = 2.5 \text{ days}$$

In general: $T_{1/2} = D \times \log_R 2$

Part 2: Containment of the COVID-19 pandemic

1. Let the students discuss in groups which value, R_0 or D, could be changed by containment action (not vaccination or medication) and what those actions might be. It should become clear that *D* is not affected, but a change to R_0 through adopting specific measures (physical distancing, face protection) could reduce the spread of the virus. This is expressed by the 'effective reproduction number', *R*.
2. Discuss with your students the value of *R* required so that the number of newly infected people no longer increases, i.e., one person can only infect a maximum of one other person. The *R* value should be a maximum of one.
3. Have the students read the background information on their worksheet about the connection between R_0 and *R* and then use the Geogebra applet (https://www.geogebra.org/m/qavutkx5) to simulate different scenarios of COVID-19 containment. The Geogebra applet looks like this:

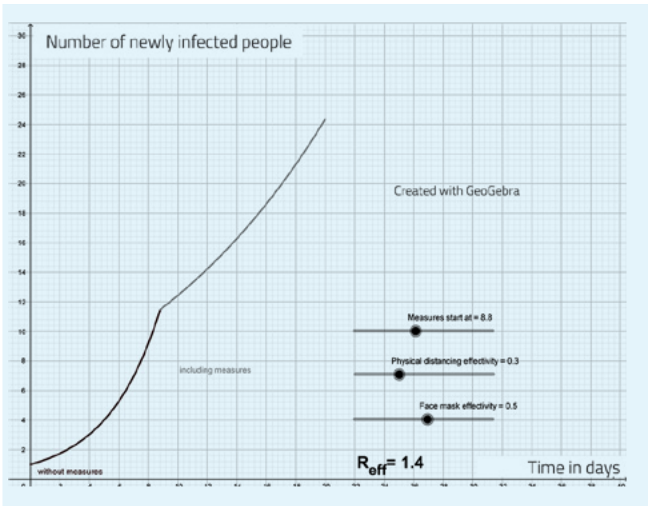

Image courtesy of Wolfgang Vieser, Copyright: © International GeoGebra Institute, 2013

4. The students should investigate the following questions:
   a. How does the timing of the containment measures change the course of the graph?
   b. How do the following containment measures affect the doubling time if put in place from the beginning?
      - only physical distancing with 50% effectivity
      - physical distancing plus use of face masks with 50% effectivity
5. Have the students calculate the doubling time for these two cases using the general formula for the doubling time. The results should be as follows:
   a. Physical distancing only: $T_{1/2} = 5 \times \log_2 2$ (twice as long as without any measures)
   b. Both measures: $T_{1/2} = 5 \times \log_1 2 = \infty$ (spread is contained)

Part 3: COVID-19 spread in a more realistic setting – herd immunity

1. Have the students discuss what additional measures can help to reduce the spread. It should emerge that the number of people who are immune (through vaccination or recovery from the disease) can change the *R* value.
2. Have the students read the background information on their worksheet about how immunisation changes the *R* value. They can then calculate the percentage of the population that needs to be immunized to contain the spread, i.e., to reduce the effective R value to one. The result should be 75%.

Discussion

Compare the percentage of the population that needs to be immunized to achieve herd immunity for COVID-19 (R_0 = 4) versus measles (R_0 = 15) and polio (R_0 = 6) and discuss with them the challenges of associated vaccination campaigns. The results are 75% (COVID-19), 93% (measles), and 83% (polio).

Resources

- Watch a video on how repeatedly folding a piece of paper could get you all the way to the moon.
- See the concept of herd immunity demonstrated using mousetraps!

AUTHOR BIOGRAPHY

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